Hands on introduction to BAT

The BAT men: Frederik Beaujean*, Allen Caldwell*, Daniel Kollár,
Kevin Kröninger, Shabnaz Pashapour
*Max-Planck-Institut für Physik, München

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Goal: Learn BAT in simple example

We will consider measuring the decay rate of a radioactive isotope, in the presence of background.

Two measurements:

1. One without source to measure the background
2. One with source present

Data Set	Run Time	Events
1	100	50
2	100	55

What do we report for the decay rate of the isotope?

$$N = N_0 e^{-t/\tau} \qquad \frac{dN}{dt} = -\frac{N}{\tau}$$

- Total rate = signal rate + background rate $R = R_S + R_B$
- Measured for a time *T* and observed N_1, N_2 events.
- Assume R_S, R_B constant. $R_S \approx \frac{N_0}{\tau_S}$

Learn about probable values of R_s using Bayes' Theorem

Mathematical Formulation I

Bayes' Theorem:

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$

Then #events, N_i, in a time window T follows a Poisson distribution. Probability of the data (likelihood) is:

$$P\left(N_1|R_B\right) = \frac{e^{-n_B} \cdot \left(n_B\right)^{N_1}}{N_1!} \qquad n_B = R_B \cdot T$$

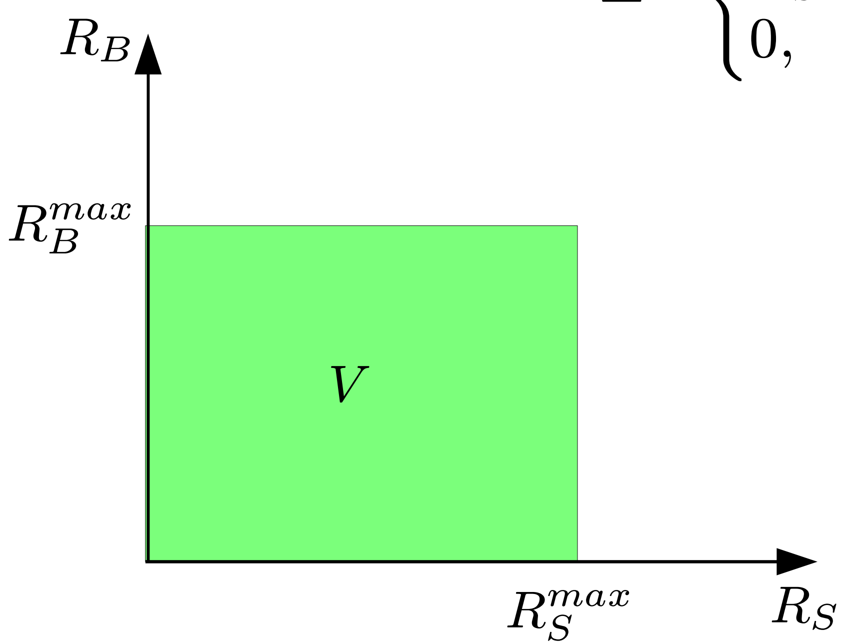
$$P\left(N_2|R_B, R_S\right) = \frac{e^{-n_{S+B}} \cdot \left(n_{S+B}\right)^{N_2}}{N_2!} \qquad n_{S+B} = \left(R_S + R_B\right) \cdot T$$

Mathematical Formulation II

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$

simplest choice of prior: flat in a box $V \equiv [0, R_S^{max}] \times [0, R_B^{max}]$

$$\begin{aligned} P(R_S, R_B) &= P(R_S)\, P(R_B) \\ &= \begin{cases} \frac{1}{R_S^{max}} \cdot \frac{1}{R_B^{max}}, & (R_S, R_B) \in V \\ 0, & \text{else} \end{cases} \end{aligned}$$



Combining the measurements

Bayes' Theorem: $\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$

I. First N_1 only, then combine with N_2

$$P(R_B|N_1) \propto P(N_1|R_B) \cdot P(R_B)$$

$$P(R_B, R_S|N_1, N_2) \propto P(N_2|R_B, R_S) \cdot P(R_S)\, P(R_B|N_1)$$

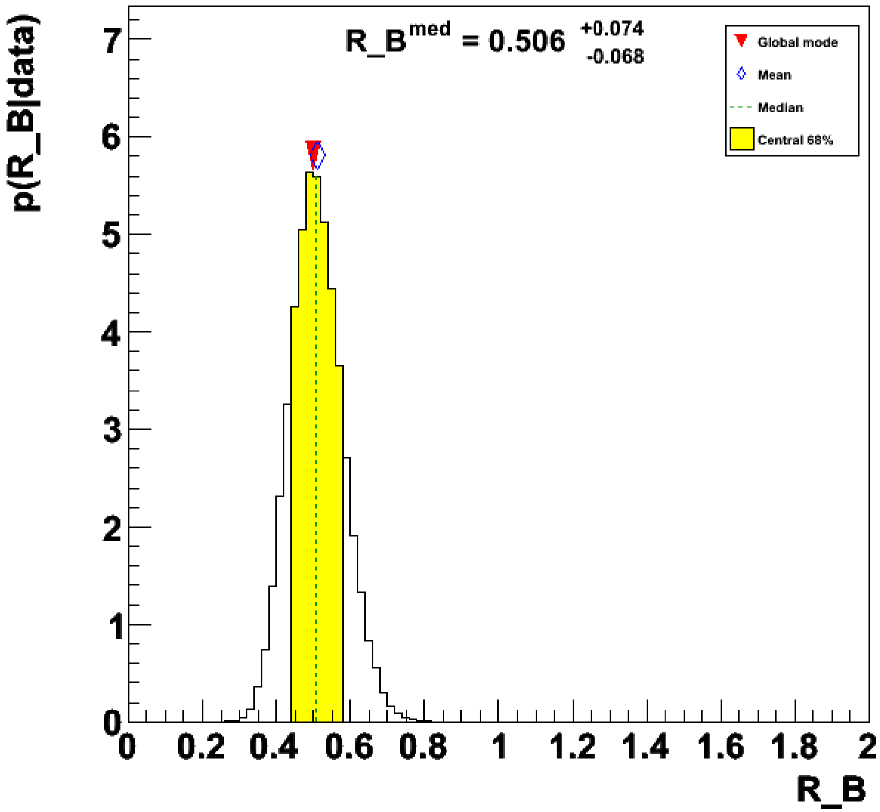
II. N_1, N_2 together

$$P(R_B, R_S|N_1, N_2) \propto P(N_2|R_B, R_S) P(N_1|R_B) \cdot P(R_S)\, P(R_B)$$

$\Rightarrow$ **Exactly the same posterior** $P(R_B, R_S|N_1, N_2)$

Results: Use only background measurement

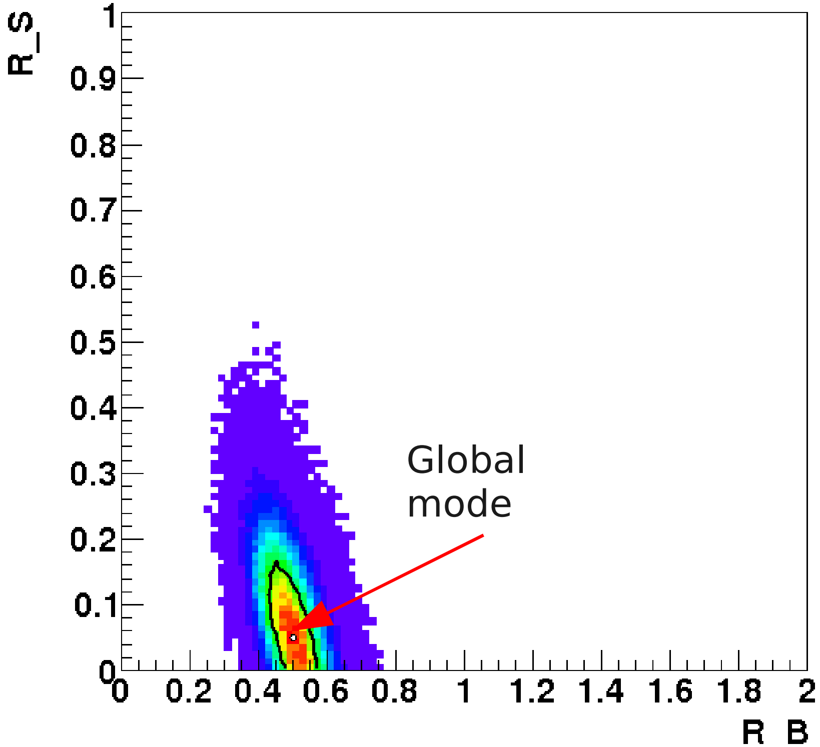
$$P\left(R_B|N_1\right) \propto P\left(N_1|R_B\right) \cdot P\left(R_B\right)$$

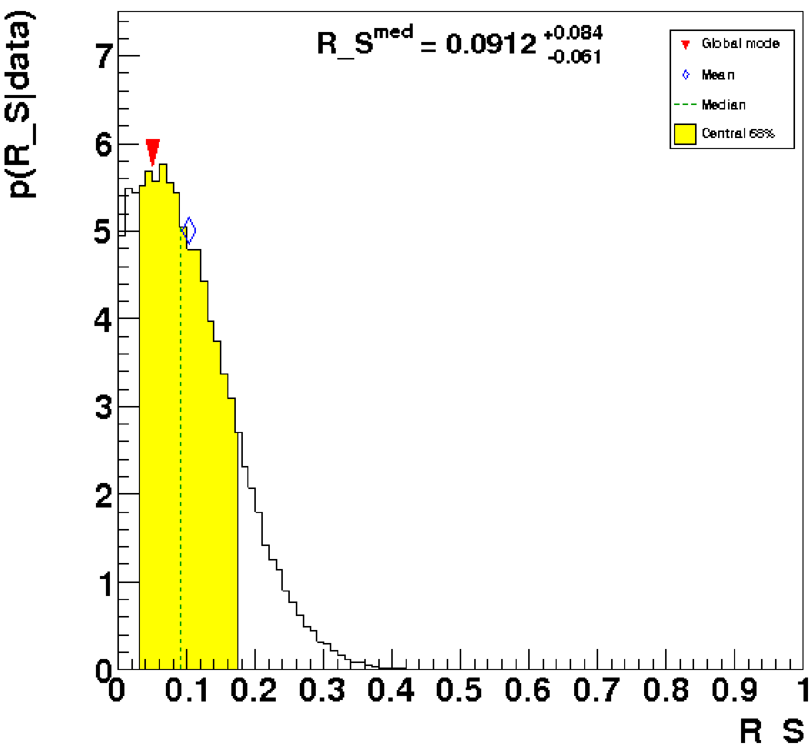


Results using N_1, N_2

$$P\left(R_B, R_S | N_1, N_2\right)$$

$$P\left(R_S | N_1, N_2\right) = \int \mathrm{d}R_B \; P\left(R_S, R_B | N_1, N_2\right)$$





...
List of parameters and global mode:
(0) Parameter "R_B": 0.5 +- 0.0706487
(1) Parameter "R_S": 0.0499999 +- 0.0987574
..

...
Parameter "R_S"
Mean +- sqrt(V): 0.1028 +- 0.07106
Median +- central 68% interval: 0.091 + 0.084 - 0.061
(Marginalized) mode: 0.065

Standard output: marginal distribution plots, text...

Links

The BAT web page

http://mpp.mpg.de/bat/

Navigate to

Documentation → Tutorials → Counting Experiment

Direct link:

http://mpp.mpg.de/bat/?page=tutorials&name=counting_experiment